Verification – Hand Calculations

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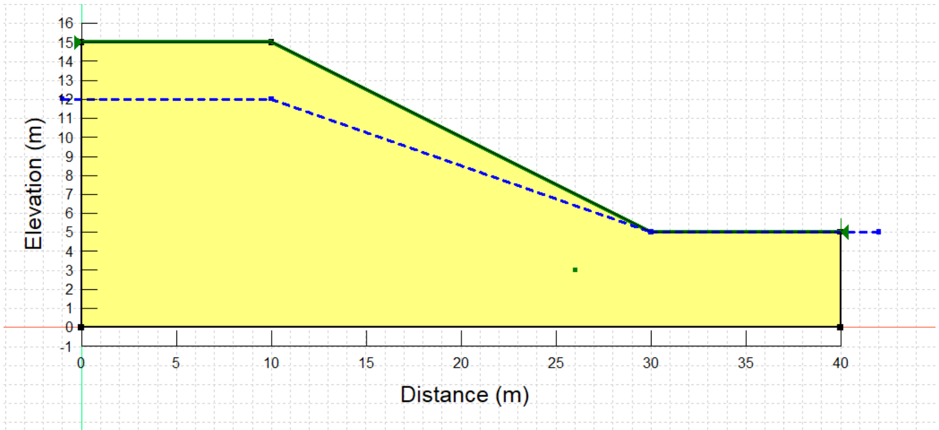
Introduction

SLOPE/W is formulated based on limit equilibrium theory. The forces acting on a sliding mass can therefore be verified using hand calculations. Moreover, the verification process can increase the analyst's confidence and understanding of the software. The objective of this example is to demonstrate the verification calculations.

Numerical Simulation

The model domain and material properties used for this illustration is shown in Figure 1. It is a simple 2:1 slope that is 10 m in height. Only one slip surface is analyzed. The problem was analyzed using the Ordinary and Bishop methods. The Ordinary method is amenable to hand-calculations because an iterative solution strategy is not required. In contrast, the Bishop method requires an iterative solution because the factor of safety and slice force equations are non-linear.



Color	Name	Model	Unit Weight (kN/m³)	Cohesion' (kPa)	Phi' (°)	Phi-B (°)	Piezometric Line
	New Material	Mohr-Coulomb	20	5	30	0	1

Figure 1. Example configuration.

Results and Discussion

The Ordinary Method

The geometric information about the sliding mass and the discretization can be copied from SLOPE/W and pasted into a spreadsheet using the Draw Graph functionality. The slice height, slice width (b), base inclination angle (α), and base pore-pressure (u) are presented in Table 1. It is noted that the pore-pressure for the two slices above the piezometric line was set to zero for expediency. The slice weight (W) and slice base length (L) were computed in the spreadsheet using the following equations:

$$W = h * b * \gamma$$ Equation 1

$$L = \frac{b}{\cos\left(\frac{\alpha\pi}{180}\right)}$$ Equation 2

The factor of safety for the Ordinary method is calculated as (Duncan and Wright, 2005):

$$FofS = \frac{\sum\left[c^{'}L + \left(W\cos\alpha - uL\cos^{2}\alpha\right)\tan\phi^{'}\right]}{\sum W\sin\alpha}$$ Equation 3

Table 2 presents the calculated values for each slice. The sum of the denominator in the factor of safety equation is 795.340 kN. The column titled 'Friction' represents the frictional component of the shear resistance along the slip surface. The column titled '$C \; x \; L$' represents the cohesive component of the shear resistance. The combination of these two (Resistance) is the numerator in the factor of safety equation. The sum of the Resistance is 930.640 kN. The factor of safety is given as:

$$FofS = \frac{930.64 \; kN}{795.64 \; kN} = 1.170 \qquad \text{Equation 4}$$

This value is identical to the result calculated by SLOPE/W presented in Figure 2.

Table 1. Geometric data for the slices.

Slice #	Height (m)	b (m)	W (kN)	Base α	Base L (m)	u (kPa)
1	0.821	0.870	14.287	-62.087	1.858	0.000
2	2.321	0.870	40.387	-57.351	1.613	0.000
3	3.801	1.234	93.778	-52.391	2.022	7.853
4	4.887	0.952	93.081	-47.702	1.415	19.204
5	5.393	0.952	102.727	-43.965	1.323	25.571
6	5.782	0.952	110.139	-40.452	1.252	30.789
7	6.072	0.952	115.666	-37.115	1.194	35.035
8	6.277	0.952	119.559	-33.920	1.148	38.441
9	6.405	0.952	122.004	-30.841	1.109	41.100
10	6.465	0.952	123.143	-27.857	1.077	43.088
11	6.462	0.952	123.088	-24.955	1.050	44.460
12	6.401	0.952	121.925	-22.119	1.028	45.263
13	6.286	0.952	119.724	-19.339	1.009	45.531
14	6.118	0.952	116.542	-16.606	0.994	45.293
15	5.902	0.952	112.424	-13.912	0.981	44.574
16	5.639	0.952	107.404	-11.248	0.971	43.390
17	5.329	0.952	101.511	-8.609	0.963	41.757
18	4.975	0.952	94.766	-5.989	0.958	39.685
19	4.577	0.952	87.183	-3.381	0.954	37.182
20	4.136	0.952	78.772	-0.780	0.952	34.252
21	3.651	0.952	69.537	1.820	0.953	30.899
22	3.123	0.952	59.477	4.423	0.955	27.120
23	2.551	0.952	48.585	7.035	0.960	22.914
24	1.935	0.952	36.851	9.663	0.966	18.273
25	1.507	0.989	29.807	12.362	1.012	14.781
26	1.265	0.989	25.019	15.140	1.024	12.406
27	0.971	0.989	19.204	17.956	1.039	9.523
28	0.623	0.989	12.317	20.818	1.058	6.108
29	0.217	0.989	4.300	23.735	1.080	2.132

Table 2. Factor of safety calculations for the Ordinary method.

Slice #	$W\ sin\ \alpha$ (kN)	$C\ x\ L$ (kN)	$W\ cos\ \alpha$ (kN)	$uL\ cos^2\alpha$	Friction (kN)	Resistance (kN)
1	12.625	9.292	6.688	0.000	3.861	13.15
2	34.005	8.063	21.789	0.000	12.580	20.64
3	74.291	10.108	57.230	5.912	29.628	39.74
4	68.847	7.076	62.643	12.309	29.060	36.14
5	71.316	6.616	73.939	17.529	32.568	39.18
6	71.460	6.258	83.810	22.313	35.505	41.76
7	69.795	5.972	92.235	26.608	37.890	43.86
8	66.718	5.738	99.212	30.380	39.741	45.48
9	62.545	5.546	104.752	33.608	41.075	46.62
10	57.542	5.386	108.872	36.281	41.911	47.30
11	51.931	5.252	111.596	38.390	42.266	47.52
12	45.908	5.140	112.952	39.935	42.156	47.30
13	39.648	5.047	112.969	40.916	41.600	46.65
14	33.307	4.969	111.681	41.337	40.613	45.58
15	27.030	4.906	109.126	41.206	39.214	44.12
16	20.951	4.855	105.341	40.530	37.418	42.27
17	15.196	4.816	100.367	39.321	35.245	40.06
18	9.887	4.788	94.248	37.589	32.712	37.50
19	5.141	4.770	87.031	35.350	29.838	34.61
20	1.072	4.762	78.764	32.618	26.642	31.40
21	-2.208	4.764	69.502	29.412	23.145	27.91
22	-4.587	4.776	59.300	25.752	19.369	24.14
23	-5.951	4.798	48.220	21.658	15.335	20.13
24	-6.185	4.830	36.329	17.156	11.069	15.90
25	-6.381	5.062	29.116	14.277	8.567	13.63
26	-6.535	5.122	24.150	11.842	7.106	12.23
27	-5.920	5.197	18.268	8.958	5.375	10.57
28	-4.377	5.290	11.513	5.645	3.388	8.68
29	-1.731	5.401	3.936	1.930	1.158	6.56
Sum	795.340					930.640

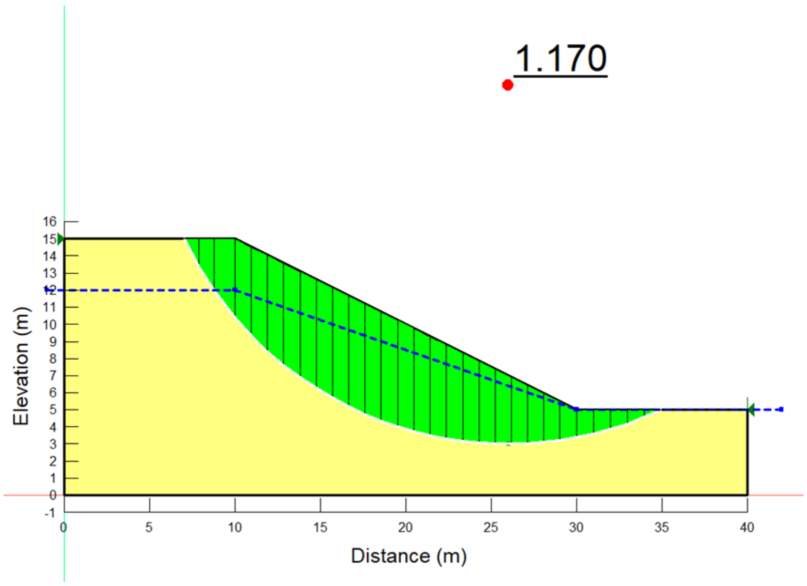


Figure 2. Ordinary method factor of safety from SLOPE/W.

Additional hand-calculations can be done by examining the details for each slice. Consider, for example, the free body diagram for Slice 5 (Figure 3). The base inclination (α) is 43.965° and the base length (L) is 1.3232 m. The total slice weight (W) is 102.73 kN and the pore-pressure (u) at the mid-point of the slice base is 25.571 kPa. The effective normal stress ($\sigma_n^{'}$) at the slice base is:

$$\sigma_n^{'} = \frac{W\cos\alpha}{L} - u\cos^2\alpha = \frac{102.73 * 0.7198}{1.3232} - (25.571 * 0.7198 * 0.7198)$$ Equation 5

$$\sigma_n^{'} = 55.8794 - 13.2475 = 42.6320\ kPa$$ Equation 6

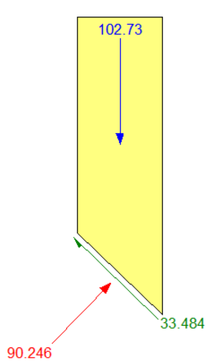


Figure 3. Free body diagram for Slice 5.

The total base normal (N) is:

$$N = (\sigma_n^{'} + u)L = (42.6320 + 25.571) * 1.3232 = 90.2462\ kN$$ Equation 7

The total shear resistance (S) at the slice base is:

$$S = c^{'}L + \sigma_n^{'}\tan\phi^{'} = 5 * 1.3232 + 42.6320 * 0.5774 = 39.1844\ kN$$ Equation 8

The portion of S that is mobilized (S_{mob}) is:

$$S_{mob} = \frac{S}{FofS} = \frac{39.1844}{1.170} = 33.48\ kN$$ Equation 9

The value is identical to that shown on the free body diagram of Slice 5 (Figure 3). It is important to note that the mobilized shear is required in the force equilibrium equations, not the shear strength.

The Bishop Method

The Bishop method considers inter-slice normal forces. Moreover, the normal force at the base of the slice is determined by summing forces in the vertical direction. As a result, the normal force N becomes a function of the factor of safety and is given by:

$$N = \frac{W - \dfrac{\left(c^{'}L\sin\alpha - uL\sin\alpha\tan\phi^{'}\right)}{F}}{\cos\alpha + \dfrac{\sin\alpha\tan\phi^{'}}{F}}$$ Equation 10

The denominator in this equation is often given the variable name m_α. Once N is known for each slice, the Bishop factor of safety can be computed from:

$$FofS = \frac{\sum\left[c^{'}L + (N - uL)\tan\phi^{'}\right]}{\sum W\sin\alpha}$$ Equation 11

The normal force in Equation 10 is a function of the factor of safety, while the factor of safety is a function of the normal force (Equation 11). An iterative solution procedure is therefore required to solve the equations. The Bishop factor of safety can be computed in a spreadsheet fairly readily. The slice weight (W), base length (L), base pore-pressure (u), and base inclination (α) are obtained from Table 1. The denominator in the factor of safety equation ($W\sin\alpha$) remains the same for all iterations. The same is true for the cohesive component of the resistance in Equation 11.

It is necessary to assume a value for the factor of safety on the first iteration. The starting value used in the hand calculations was the Ordinary method factor of safety. Table 3 shows the calculations when the trial factor of safety is 1.170. The new computed factor of safety is the total resistance divided by the total driving force ($W \sin \alpha$) and is equal to 1.236. The forces are updated and the factor is safety is calculated three more times (Table 4 to Table 6) until the factor of safety is equal to:

$$F = 993.173/795.3401 = 1.249 \quad \text{Equation 15}$$

This is the same value computed by SLOPE/W (Figure 4).

Table 3. Calculations with a trial factor of safety equal to 1.170

Slice #	$W \sin \alpha$ **(kN)**	C **(kN)**	m_α	N **kN**	**Friction (kN)**	**Total (M+N)**
1	12.6250	9.2922	0.9042	8.039	4.642	13.934
2	34.0052	8.0629	0.9550	36.214	20.908	28.971
3	74.2909	10.1078	1.0012	93.030	44.546	54.653
4	68.8469	7.0757	1.0380	94.921	39.113	46.188
5	71.3156	6.6160	1.0623	103.916	40.461	47.077
6	71.4599	6.2579	1.0811	110.078	41.306	47.564
7	69.7952	5.9716	1.0952	114.177	41.762	47.734
8	66.7177	5.7385	1.1052	116.696	41.903	47.641
9	62.5454	5.5461	1.1116	117.947	41.776	47.322
10	57.5416	5.3861	1.1147	118.144	41.413	46.799
11	51.9307	5.2522	1.1148	117.432	40.835	46.087
12	45.9083	5.1402	1.1122	115.910	40.056	45.196
13	39.6481	5.0467	1.1070	113.647	39.081	44.128
14	33.3072	4.9692	1.0993	110.684	37.914	42.884
15	27.0299	4.9058	1.0893	107.044	36.552	41.458
16	20.9506	4.8552	1.0770	102.735	34.988	39.843
17	15.1961	4.8162	1.0626	97.747	33.212	38.028
18	9.8874	4.7880	1.0460	92.058	31.209	35.997
19	5.1413	4.7702	1.0274	85.632	28.959	33.729
20	1.0720	4.7623	1.0066	78.416	26.438	31.200
21	-2.2081	4.7643	0.9838	70.342	23.614	28.378
22	-4.5867	4.7761	0.9590	61.322	20.447	25.224
23	-5.9508	4.7980	0.9320	51.242	16.890	21.688
24	-6.1853	4.8304	0.9030	39.959	12.878	17.708
25	-6.3812	5.0616	0.8712	33.464	10.681	15.743
26	-6.5345	5.1221	0.8364	29.321	9.591	14.713
27	-5.9204	5.1974	0.7992	23.859	8.060	13.258
28	-4.3774	5.2896	0.7593	16.844	5.995	11.284
29	-1.7306	5.4011	0.7168	7.952	3.262	8.663
Totals	795.3401					983.091

Table 4. Calculations with a trial factor of safety equal to 1.236.

Slice #	$W \sin \alpha$ (kN)	C (kN)	m_α	N kN	Friction (kN)	Total (M+N)
1	12.6250	9.2922	0.8809	8.678	5.010	14.302
2	34.0052	8.0629	0.9328	37.409	21.598	29.661
3	74.2909	10.1078	0.9803	95.048	45.710	55.818
4	68.8469	7.0757	1.0185	96.453	39.997	47.073
5	71.3156	6.6160	1.0440	105.346	41.286	47.902
6	71.4599	6.2579	1.0640	111.403	42.071	48.328
7	69.7952	5.9716	1.0793	115.397	42.466	48.438
8	66.7177	5.7385	1.0905	117.810	42.546	48.284
9	62.5454	5.5461	1.0981	118.957	42.358	47.905
10	57.5416	5.3861	1.1024	119.050	41.936	47.322
11	51.9307	5.2522	1.1037	118.237	41.300	46.552
12	45.9083	5.1402	1.1023	116.615	40.463	45.603
13	39.6481	5.0467	1.0983	114.255	39.432	44.479
14	33.3072	4.9692	1.0918	111.197	38.211	43.180
15	27.0299	4.9058	1.0830	107.465	36.795	41.701
16	20.9506	4.8552	1.0719	103.066	35.179	40.035
17	15.1961	4.8162	1.0587	97.993	33.354	38.170
18	9.8874	4.7880	1.0433	92.222	31.304	36.092
19	5.1413	4.7702	1.0258	85.720	29.010	33.780
20	1.0720	4.7623	1.0063	78.435	26.449	31.211
21	-2.2081	4.7643	0.9847	70.301	23.590	28.354
22	-4.5867	4.7761	0.9610	61.229	20.394	25.170
23	-5.9508	4.7980	0.9353	51.112	16.815	21.613
24	-6.1853	4.8304	0.9074	39.809	12.792	17.622
25	-6.3812	5.0616	0.8768	33.288	10.580	15.642
26	-6.5345	5.1221	0.8433	29.113	9.471	14.593
27	-5.9204	5.1974	0.8073	23.628	7.927	13.124
28	-4.3774	5.2896	0.7687	16.606	5.857	11.147
29	-1.7306	5.4011	0.7274	7.733	3.135	8.536
Totals	795.3401					991.636

Table 5. Calculations with a trial factor of safety equal to 1.247.

Slice #	$W \sin \alpha$ (kN)	C (kN)	m_α	N kN	Friction (kN)	Total (M+N)
1	12.6250	9.2922	0.8773	8.778	5.068	14.360
2	34.0052	8.0629	0.9294	37.596	21.706	29.769
3	74.2909	10.1078	0.9771	95.363	45.893	56.000
4	68.8469	7.0757	1.0155	96.692	40.135	47.211
5	71.3156	6.6160	1.0412	105.568	41.415	48.031
6	71.4599	6.2579	1.0614	111.609	42.190	48.447
7	69.7952	5.9716	1.0768	115.586	42.575	48.547
8	66.7177	5.7385	1.0882	117.982	42.645	48.384
9	62.5454	5.5461	1.0960	119.113	42.449	47.995
10	57.5416	5.3861	1.1005	119.190	42.017	47.403
11	51.9307	5.2522	1.1020	118.361	41.371	46.624
12	45.9083	5.1402	1.1008	116.724	40.526	45.666
13	39.6481	5.0467	1.0969	114.348	39.486	44.533
14	33.3072	4.9692	1.0906	111.275	38.256	43.225
15	27.0299	4.9058	1.0820	107.529	36.832	41.738
16	20.9506	4.8552	1.0711	103.117	35.209	40.064
17	15.1961	4.8162	1.0581	98.030	33.376	38.192
18	9.8874	4.7880	1.0429	92.248	31.318	36.106
19	5.1413	4.7702	1.0256	85.734	29.018	33.788
20	1.0720	4.7623	1.0062	78.438	26.451	31.213
21	-2.2081	4.7643	0.9848	70.294	23.586	28.350
22	-4.5867	4.7761	0.9613	61.215	20.386	25.162
23	-5.9508	4.7980	0.9358	51.092	16.803	21.601
24	-6.1853	4.8304	0.9081	39.786	12.778	17.609
25	-6.3812	5.0616	0.8777	33.262	10.565	15.626
26	-6.5345	5.1221	0.8443	29.081	9.452	14.575
27	-5.9204	5.1974	0.8085	23.593	7.906	13.104
28	-4.3774	5.2896	0.7701	16.570	5.836	11.126
29	-1.7306	5.4011	0.7290	7.700	3.116	8.517
Totals	795.3401					992.967

Table 6. Calculations with a trial factor of safety equal to 1.248.

Slice #	$W \sin \alpha$ (kN)	C (kN)	m_α	N kN	Friction (kN)	Total (M+N)
1	12.6250	9.2922	0.8773	8.778	5.068	14.360
2	34.0052	8.0629	0.9294	37.596	21.706	29.769
3	74.2909	10.1078	0.9771	95.363	45.893	56.000
4	68.8469	7.0757	1.0155	96.692	40.135	47.211
5	71.3156	6.6160	1.0412	105.568	41.415	48.031
6	71.4599	6.2579	1.0614	111.609	42.190	48.447
7	69.7952	5.9716	1.0768	115.586	42.575	48.547
8	66.7177	5.7385	1.0882	117.982	42.645	48.384
9	62.5454	5.5461	1.0960	119.113	42.449	47.995
10	57.5416	5.3861	1.1005	119.190	42.017	47.403
11	51.9307	5.2522	1.1020	118.361	41.371	46.624
12	45.9083	5.1402	1.1008	116.724	40.526	45.666
13	39.6481	5.0467	1.0969	114.348	39.486	44.533
14	33.3072	4.9692	1.0906	111.275	38.256	43.225
15	27.0299	4.9058	1.0820	107.529	36.832	41.738
16	20.9506	4.8552	1.0711	103.117	35.209	40.064
17	15.1961	4.8162	1.0581	98.030	33.376	38.192
18	9.8874	4.7880	1.0429	92.248	31.318	36.106
19	5.1413	4.7702	1.0256	85.734	29.018	33.788
20	1.0720	4.7623	1.0062	78.438	26.451	31.213
21	-2.2081	4.7643	0.9848	70.294	23.586	28.350
22	-4.5867	4.7761	0.9613	61.215	20.386	25.162
23	-5.9508	4.7980	0.9358	51.092	16.803	21.601
24	-6.1853	4.8304	0.9081	39.786	12.778	17.609
25	-6.3812	5.0616	0.8777	33.262	10.565	15.626
26	-6.5345	5.1221	0.8443	29.081	9.452	14.575
27	-5.9204	5.1974	0.8085	23.593	7.906	13.104
28	-4.3774	5.2896	0.7701	16.570	5.836	11.126
29	-1.7306	5.4011	0.7290	7.700	3.116	8.517
Totals	795.3401					992.967

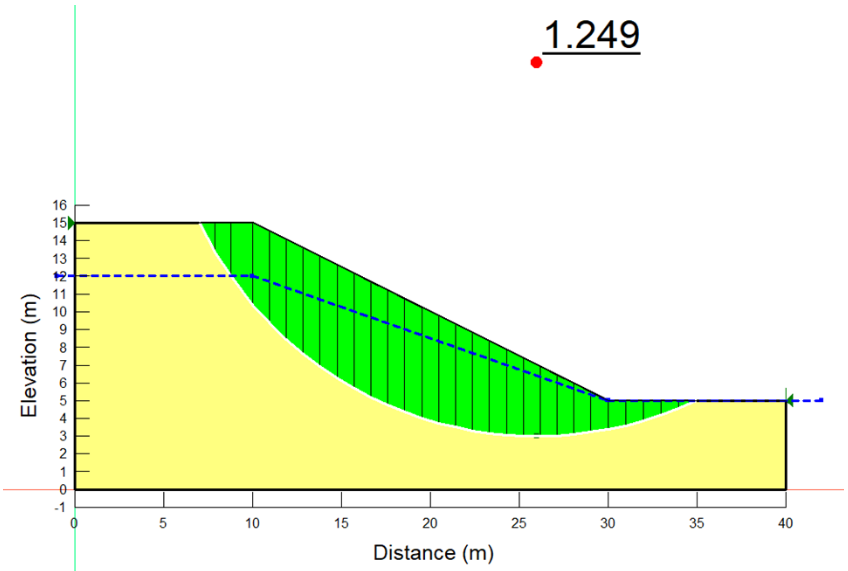


Figure 4. Bishop method factor of safety from SLOPE/W.

Figure 5 shows the free body diagram for Slice 5. The hand-calculated normal force for Slice 5 in Table 6 is 105.03 kN (based on the 2nd last trial $FofS$), which is the same as shown in Figure 5 (based on a factor of safety equal to 1.249). The total resistance in Table 6 is 48.051 kN. Dividing this by the factor of safety 1.248 gives 38.50 kN, which is the mobilized shear shown on slice 5.

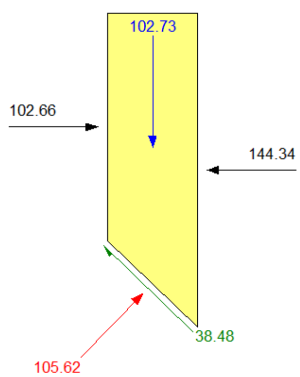


Figure 5. Slice 5 for the Bishop analysis.

Summary and Conclusions

This example demonstrates how spreadsheet calculations can be used to verify the SLOPE/W computed values. This type of exercise helps to heighten our understanding of the limit equilibrium formulation.

References

Duncan, J.M and Wright, S.G. 2005. Soil Strength and Slope Stability. John Wiley & Sons, Inc., p. 67.