Add-In – Conductive and Radiative Heat Transfers

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Introduction

Most analyses of heat transfer in soils neglect the effect of heat radiation that occurs as electromagnetic waves propagate across pore-air spaces. Although very complex, radiative heat transfer is greatly simplified by considering the pore air as a gray and diffuse medium with wavelength and direction independent properties. In so doing, the soil particles are exposed only to the radiation emitted by their neighbors, and radiative heat transfer can be expressed as a diffusion equation. Although most prevalent at elevated temperatures, this type of heat transfer also occurs in coarse-grained soils under atmospheric conditions. As recently highlighted by Fillion et al. (2011), the effective (combined conductive and radiative) thermal conductivity of coarse-grained soils can be two to three times greater than that ascribed to conduction alone. The radiative component of effective thermal conductivity also increases with increasing particle diameter. Hence, the failure to account for radiative heat transfer may lead to significant underestimation of soil temperatures, and incorrect design of modern engineered structures.

The objective of this example is to verify a conductive and radiative material model Add-In using an analytical solution for diffusive heat transfer in a homogeneous semi-infinite soil column. The verification exercise lends credibility to the use of the Add-In for problems involving conductive and radiative heat transfers in soils.

Background

For the sake of simplicity, the pore-air is herein considered as a gray, emitting, and adsorbing medium in which radiation travels only a short distance before being absorbed. The soil particles are thus exposed only to the radiation emitted by their neighbors, and radiative heat transfer is expressed as a diffusion equation. In so doing, the mathematical description of conductive and radiative heat transfers hinges on the combination of the principle of conservation of energy and the phenomenological equations that describe heat conduction and radiation, which leads to the following equation:

$$\nabla \cdot \left[k_t^e \nabla T\right] = C_t \frac{\partial T}{\partial t} \quad \text{Equation 1}$$

where $k_t^e = k_t^c + k_t^r$ is the effective thermal conductivity, k_t^c is the conductive component of effective thermal conductivity, k_t^r is the radiative component of effective thermal conductivity, T is the temperature, C_t is the volumetric heat capacity of the soil at constant pressure, and t is the time. The radiative component of effective thermal conductivity is often derived from the analytical solution for radiative heat transfer between plates with similar surface emissivity, which reads:

$$q_r = -4\left(\frac{\varepsilon}{2-\varepsilon}\right) D\sigma \bar{T}^3 \frac{\partial T}{\partial t} \quad \text{Equation 2}$$

where q_r is the radiative heat flux, ε is the surface emissivity, D is the plate separation which is set equal to the particle diameter, σ is the Stefan-Boltzmann constant, and $\bar{T}$ is the average absolute temperature. Although quite accurate, this equation is often criticized for neglecting the effect of long-range radiation. The pseudo-continuous model circumvents this limitation by representing the soil as a random assembly of solid particles, and solving the radiative heat transfer equation as if the medium were a continuum. In the case of a parallel-plane layer of absorbing, emitting, and scattering medium, the solution reads (Chen and Churchill, 1963):

$$q_r = -4\left[\frac{2}{(a+2b)D}\right] D\sigma \bar{T}^3 \frac{\partial T}{\partial t} \quad \text{Equation 3}$$

where a and b are the absorption and scattering coefficients, respectively. Although these coefficients can be determined experimentally, Vortmeyer (1978) established a theoretical relationship between the pseudo-continuous and discontinuous models that leads to explicit expressions for the absorption and scattering coefficients. Inserting these expressions into Equation 3 results in the following expression for radiative heat flux:

$$q_r = -4\left[\frac{\varepsilon}{2-\varepsilon} + \frac{2L}{(2-\varepsilon)(1-L)}\right] D\sigma \bar{T}^3 \frac{\partial T}{\partial t} \quad \text{Equation 4}$$

where L is the long-range radiation transmission parameter. Given its proven ability to describe radiative heat flux (Vortmeyer, 1978; Tien, 1988), the equation is adopted in this example and the radiative component of effective thermal conductivity is expressed as:

$$k_t^r = 4\left[\frac{\varepsilon}{2-\varepsilon} + \frac{2L}{(2-\varepsilon)(1-L)}\right]D\sigma\bar{T}^3 \quad \text{Equation 5}$$

The surface emissivity of igneous, sedimentary, and metamorphic rocks can be taken equal to 0.925, 0.966, and 0.952, respectively (Salisbury and D'Aria, 1992). The conductive component of effective thermal conductivity is herein expressed as follows (Mottaghy and Rath, 2006):

$$k_t^c = \begin{cases} \left(k_{t,u}^{\,c} - k_{t,f}^{\,c}\right)e^{-\left(\frac{T - T_L}{w}\right)^2} + k_{t,f}^{\,c} & T < T_L \\ k_{t,u}^{\,c} & T \geq T_L \end{cases} \quad \text{Equation 6}$$

where $k_{t,u}^{\,c}$ is the unfrozen conductive component of effective thermal conductivity, $k_{t,f}^{\,c}$ is the frozen conductive component of effective thermal conductivity, T_L is the liquidus (or fusion) temperature, $w \approx (T_L - T_s)/2$ is a curve shape parameter, and T_s is the solidus (or solidification) temperature. This form of equation is also used to describe the unfrozen volumetric water content function, which enters into the Full Thermal Material Model encompassed by the Add-In.

Numerical Simulation

The ability of the material model Add-In is evaluated with a benchmark problem for diffusive heat transfer in a homogeneous semi-infinite soil column. For numerical purposes, the height of the soil column is limited to 10 m. Although the temperature is initially constant at 20°C, it suddenly rises to 40°C at the soil surface. The conductive component of effective thermal conductivity is equal to 0.0002 kJ/s/m/°C, and the volumetric heat capacity is equal to 1000 kJ/m^3/°C. The emissivity of the generic rockfill material, with an average particle diameter of 0.063 m, is set equal to an average value of 0.924, whereas the long-range radiation transmission parameter is set equal to zero. The analytical solution to this problem reads as follows (Carslaw and Jaeger, 1959):

$$T(y,t) = T_o + \Delta T erfc\left[\frac{y - y_o}{2\sqrt{\left(k_t^e/C_t\right)t}}\right] \quad \text{Equation 7}$$

where T is the temperature, y is the elevation, T_o is the initial temperature, ΔT is the change in surface temperature, y_o is the elevation at the soil surface, k_t^e is the effective thermal conductivity of the soil, C_t is the volumetric heat capacity of the soil, and t is the time. In the presence of radiative heat transfer, the effective thermal conductivity is a function of temperature and the solution must be transformed into a root-finding problem by moving all terms to the left-hand side of the equation.

In this example, the soil thermal properties are described by the Radiative Heat Transfer Add-In. Access to the Add-In functions is provided through the Add-In Function Type menu in the pop-up window of each function of the Full Thermal Material Model. The inputs of the thermal conductivity versus Temperature Add-In function are the unfrozen thermal conductivity, the frozen thermal conductivity, the liquidus temperature, the surface emissivity, the particle diameter, the long-range radiation transmission parameter, and parameter *w*. The inputs of the unfrozen volumetric water content function are the liquidus temperature, and parameter *w*.

Results and Discussion

Figure 1a shows the results of the diffusive heat transfer problem without radiative heat transfer. In this case, the increase in surface temperature slowly leads to increases in soil temperature with depth. As expected, the results are generally in very close agreement with the analytical solution. Figure 1b shows the results of the diffusive heat transfer problem with both conductive and radiative heat transfers. In this case, the effective thermal conductivity is 1.5 to 2 times larger than that ascribed to conduction. As a result, the soil temperature increases much more rapidly with time. Once again, the results are in very close agreement with the analytical solution.

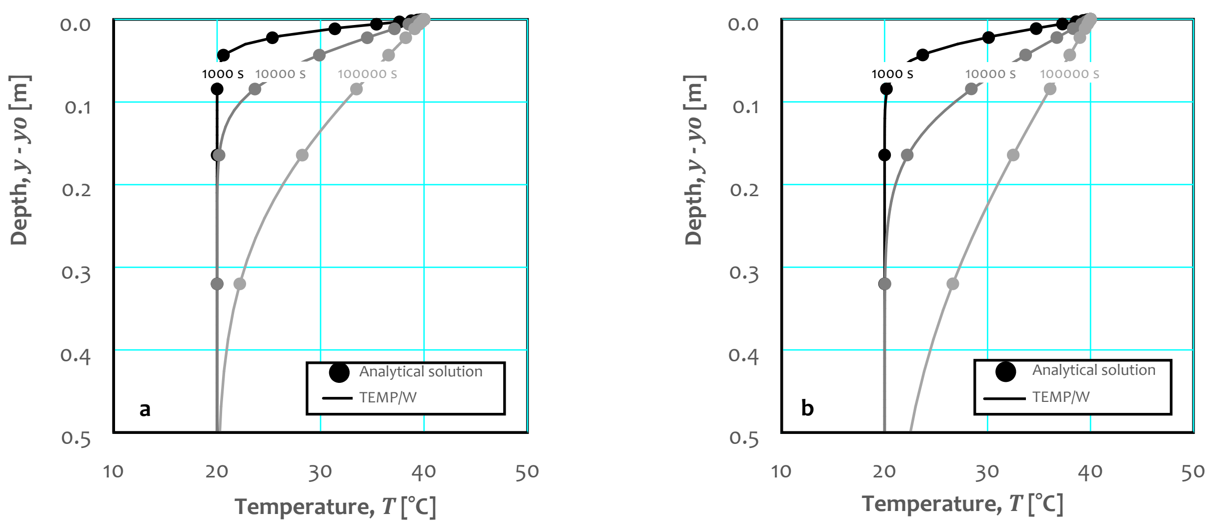

Figure 2. Temperature profile. (a) Conductive heat transfer. (b) Conductive and radiative heat transfers.

Summary and Conclusions

The objective of this example was to demonstrate the ability to solve conductive and radiative heat transfer problems with TEMP/W. The capabilities of the model were assessed with an analytical solution for diffusive heat transfer in a semi-infinite soil column. The radiative heat transfer was shown to influence the transmission of heat within the soil profile, and result in higher temperatures than those observed with conduction alone. The results were generally shown to be in very close agreement with those of the analytical solution.

References

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